Sample Problems and Solutions

Math County Invitational
MATH TOURNAMENT
For Grades 4-6

Math County Invitational
MATH TOURNAMENT
For Grades 6-8

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MATH TOURNAMENT

For Grades 4-6

Some Problems From 2008

2008 Individual Event

2. What missing number makes the statement true? $44 \times 32 = 64 \times \square$

6. The area of a square is less than 200 sq cm. The length in centimeters of each side is a counting number. What is the greatest perimeter that the square can have, in cm?

9. Amy plays Zane in a game with twelve rounds. In each round, the winner scores 5 points and the loser scores 3 points. At the end of the game, Zane's total score is 44 points. How many rounds did Amy win?

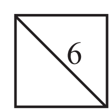
The individual event contained 10 problems.

2008 Team Event

11. Find the value: $(6 - 12) \times (6 - 9) \times (6 - 6) \times (6 - 3)$.

15. What missing number makes the statement true? $\frac{3}{5} + 17 = \frac{\square}{15} + 16$

19. In the square shown, the length of the diagonal is 6 cm. What is the area of the square, in square centimeters?



The team event contained 10 problems.

2008 Tiebreaker Event

21. Brionna walks exactly 5 blocks in 7 minutes and 30 seconds. At this rate, what is the total number of blocks that she walks in 12 minutes?

5 problems are provided to break ties.

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Sample Solutions

2008 Individual Event

2. **<u>22</u>** Doubling the 32 and halving the 44 does not change the product, nor does switching the positions of two numbers multiplying each other.

6. **<u>56</u>** Since $10^2 = 100$ and $20^2 = 400$, the side-length of the square is between 10 and 20. The side-length is less than 15 cm because $15^2 = 225$. The area of the square is $14^2 = 196$, the side-length is 14 cm and the perimeter is 56 cm.

9. **<u>8</u>** **METHOD 1:** Think of the scoring this way: each player gets 3 points per round just for playing and the winner of the round receives a bonus of 2 points. If we remove those 3 points per round from each person, then the only scoring is 2 points per round for the winner. For the whole game each player would have scored 36 fewer points, so Zane would have scored 44 – 36 = 8 points. At 2 points per round, he would have won 4 rounds and Amy would have won 8 rounds.

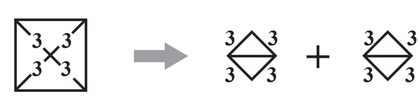
   METHOD 2: If Zane won every round, the final score would have been 60-36 in his favor. Each round he loses lowers his total score by 5 – 3 = 2 points. However, his total score was 44 points, which is 16 points less than 60 points. He lost (and Amy won) 16 ÷ 2 = 8 rounds.

2008 Team Event

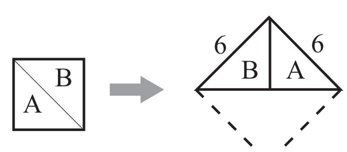
11. **<u>0</u>** In the third set of parentheses, the value is 0. The product of 0 and any number is 0.

15. **<u>24</u>** $\frac{\square}{15} + 16$: $\frac{3}{5} + 17 = \frac{9}{15} + 17 = \frac{9}{15} + (\frac{15}{15} + 16) = (\frac{9}{15} + \frac{15}{15}) + 16 = \frac{24}{15} + 16$. The numerator is 24.

19. **<u>18</u>** **METHOD 1:** Draw the other diagonal and move the four triangles as shown to form two small squares. Then the sum of the areas of the two small squares, 9 + 9, equals the area of the original square, 18 sq cm.



   METHOD 2: Rearrange the two given triangles as shown and draw the two dashed lines to complete the square.The area of the square is 6 × 6 = 36 sq cm. The area of the original square is equal to the upper half of the diagram, 18 sq cm.



2008 Tiebreaker Event

T1. **<u>8</u>** At that rate she walks 10 blocks in 15 minutes, which is 2 blocks every 3 minutes. Thus in 12 = 4 × 3 minutes, she walks 4 × 2 = 8 blocks.

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For Grades 6-8

Some Problems from 2009

2009 Individual Event

3. In lowest terms, what is the value of $\frac{180}{30} - \frac{180}{45}$?

5. What is the least positive integer that is divisible by each of 1, 2, 3, 4, 5, and 6?

10. The average of three numbers is $-\frac{1}{2}$. Two of the numbers are $\frac{1}{3}$ and $\frac{1}{4}$. What is the third number in lowest terms?

The individual event contained 10 problems

2009 Team Event

13. Using only the digits 1, 2, and 3, Julia lists all the possible three-digit numbers. In a given number the same digit can appear more than once. She chooses one of the numbers at random. What is the probability that the number she chooses is a multiple of **1**?

15. The mode of four numbers is 11, the median is 12, and the mean is 13.What is the greatest of the four numbers?

19. If a and b are positive integers, and $\frac{a}{5} + \frac{b}{7} = \frac{31}{35}$, what is the value of a and what is the value of b?

The team event contained 10 problems

2009 Tiebreaker Event

Rachel has 6 stamps. Their values are 1¢, 2¢, 4¢, 8¢, 16¢, and 32¢. She uses 5 of these stamps. How many different sums are possible?

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FOR GRADES 6-8

SAMPLE SOLUTIONS

2009 Individual Event

3. **2**. $\frac{180}{30} - \frac{180}{45} = 6 - 4 = 2$.

5. **60**. **METHOD 1:** Express each divisor in terms of its prime factors: LCM(6,5,4,3,2,1) = LCM(2×3,5,2×2,3,2,1) = $5^1 \times 3^1 \times 2^2 \times 1 = 60$.

   METHOD 2: Find the Least Common Multiples in pairs, starting with 6. LCM(6,5) = 30. Then LCM(30,4) = 60. Since 60 is also a multiple of 3, 2, and 1, the least positive integer divisible by all the given is 60.

10. **$-\frac{25}{12}$ *or* $-2\frac{1}{12}$**. For the average to be $-\frac{1}{2}$, the sum of the three numbers must be $-\frac{3}{2}$. The sum of two of the three numbers, $\frac{1}{3}$ and $\frac{1}{4}$, is $\frac{7}{12}$. To find the third number, subtract $\frac{7}{12}$ from $-\frac{3}{2}$. The third number is $-\frac{25}{12}$. (*Note: A common algebraic solution uses the above reasoning.*)

2009 Team Event

13. **$\frac{3}{27}$ *or* $\frac{1}{9}$:** For each of the three digits that can occupy the hundreds place, there are three digits that can occupy the tens place. For each of these 9 permutations (arrangements), there are 3 digits that can occupy the units place. There are a total of 27 different 3-digit numbers that may be written. Using the test of divisibility for 1 1, only 121, 132, and 231 are multiples of 11. The probability of choosing a 3-digit multiple of 1 1 is $\frac{3}{27}$ or $\frac{1}{9}$.

15. **17:** With four numbers, the mode appears two or three times: 1 1, 11, __, __. The median, 12, is the average of the middle two terms, making the third term 13. Thus, the mode must appear twice. The mean is 13, so that the sum of the four terms is 52. The fourth term is 52 – (1 1 + 11 + 13) = 17.

19. ***a* = 3 and *b* = 2:** Express all terms with a common denominator: $\frac{7a}{35} + \frac{5b}{35} = \frac{31}{35}$.
    Multiply both sides of the equation by 35: $7a + 5b = 31$
    Then a is at least 1, and, since $31 \div 7 < 5$, a is at most 4.
    In $7a + 5b = 31$, replace a by 1 through 4, inclusive, and solve for b. If $a = 1$, 2, or 4, b is not an integer. If $a = 3$, then $b = 2$.
    (*Note: This is a linear Diophantine equation, one equation with two variables and integral solutions.*)

2009 Tiebreaker Event

6. **METHOD 1:** There are two considerations. Choosing exactly 5 of these stamps is equivalent to omitting the remaining stamp, and there is no way that any two different combinations of stamps can produce the same total value. By omitting any 1 of the 6 stamps, ***six*** different sums are possible.

METHOD 2: Listing every set of 5 stamps and adding, we get 31¢, 47¢, 55¢, 59¢, 61¢, and 62¢. Six sums are possible.